Bi-Scale Porous Structures

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Abstract

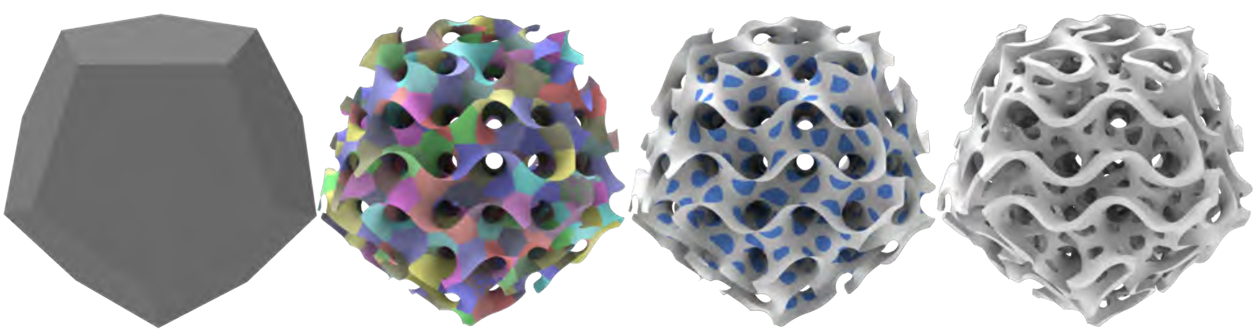
Porous structures are ubiquitous in nature and widely used for many applications. Besides the functionalities, the porous structures have considerable aesthetic appeal. In this paper, we propose a modeling framework to automatically generate bi-scale porous structures. We employ the minimal surfaces to control the large scale of the porous structures. Smooth B-Spline curves are then sculpted on the surface, such that the fine scale pores are achieved. Both the large and fine scale structures are controlled by parameters. Results can be directly fabricated and of aesthetics.

Introduction

Porous structures are of both functionality and aesthetics, which have drawn lots of attention in recent years. In order to achieve specific physical properties such as permeability, dispersion, resistivity, etc., researches try to control geometric characteristics of porous structures, like locations, patterns, and density of pores. However, it is still a tough task for designing irregular porous structures.

Minimal surfaces are defined as surfaces with zero mean curvature. I.e, at every point, the curvatures of the two maximal normal sections are equal but have opposite signs. This equilibrium of the curvatures is the basis for the aesthetic charm of minimal surfaces. Triply periodic minimal surface (TPMS) is a subset of minimal surfaces that extend periodically and indefinitely in space [1].

Our work draws inspiration from the Voronoi structure and TPMS. We aim at combining the merits of minimal surfaces and surface sculpting to generate the bi-scale porous structures.



(a) (b) (c) (d)

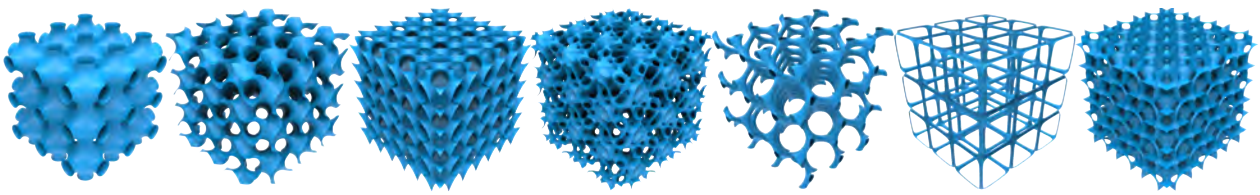
Figure 1: *Pipeline for generating the bi-scale porous structure. (a) A given dodecahedron shape; (b) the corresponding centroidal Voronoi tessellation on TPMS bounded by dodecahedron; (c) smoothly hollowed surface; (d) the fabricable, solid cast with uniform thickness.*

Overview

We propose a modeling framework to generate a bi-scale structure that is aesthetic and light-weighted. The TPMS is taken as the basic porous structure in a large scale, hollowed by B-Spline controlled smooth pores in a fine scale. The algorithm pipeline is illustrated in Fig. 1. We first take a closed manifold mesh as the input boundary and generate the TPMS defined by an implicit function in the shape. Then we compute the centroidal Voronoi tessellation (CVT) on the TPMS. Taking the Voronoi vertices as control points of a closed B-Spline curve, we create a smooth hole in each Voronoi cell. After uniform extrusion, the bi-scale porous structure can be fabricated by standard 3D printers.

Technical Details

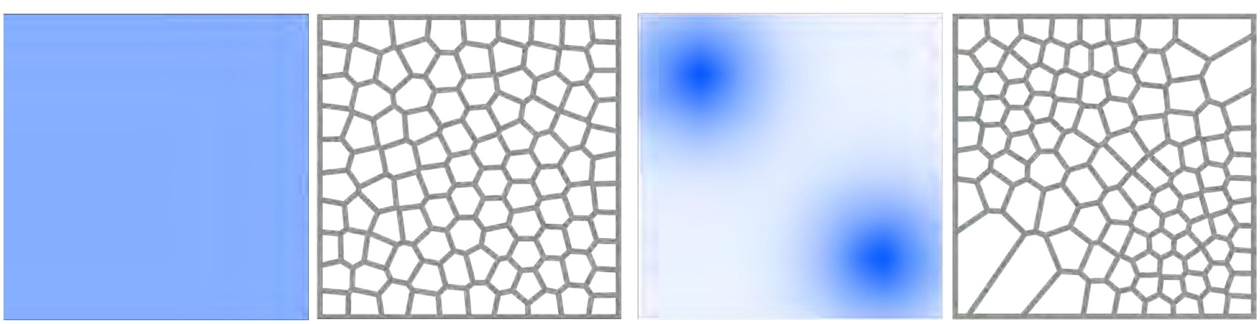
TPMS construction. A TPMS is a periodic implicit surface defined independently in three orthogonal directions. It defines a large family of surfaces which have a closed form implicit representation that results from convolving sin and cos terms along x, y, and z-axes in Euclidean space. The implicit function $\mathcal{F}(\mathbf{x}) = 0$ of TPMS can be generally expressed as $\mathcal{F}(\mathbf{x}) = \sum_{k=1}^{K} A_k \cos[2\pi(h_k \cdot \mathbf{x})/\lambda_k + p_k] - C$, where λ_k is the periodic wavelength, p_k is the phase offset, A_k is the amplitude factor, and C is a constant. The TPMS is mainly controlled by the four parameters A_k, h_k,λ_k, and p_k. Fig. 2 shows different types of TPMS.



(a) (b) (c) (d) (e) (f) (g)

Figure 2: *TPMS family. (a) P, (b) G, (c) D, (d) L, (e) Tubular-G, (f) Tubular-P, (g) FDR.*

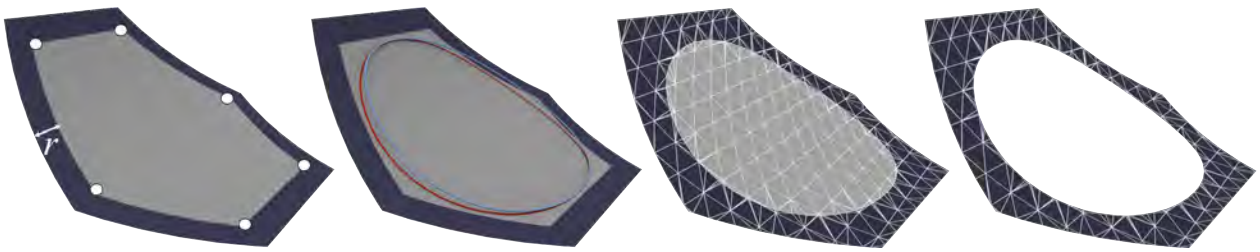
CVT-governed distribution. We adopt a hollowing operation on the TPMS to generate the bi-scale porous structure. Denote the bounded TPMS by S, we compute the centroidal Voronoi tessellation (CVT) restricted on S [2]. Let $X = (\mathbf{x}_i)_{i=1}^{n}$ be an ordered set of n sites on S. The Voronoi cell for $\mathbf{x}_i$ is defined by $\psi_i = \{\mathbf{x} \in S | d(\mathbf{x}, \mathbf{x}_i) < d(\mathbf{x}, \mathbf{x}_j), j \neq i\}$, where $d(\cdot)$ denotes the Euclidean distance. The CVT energy function is $E(X) = \sum_{i=1}^{n} \int_{\psi_i} \rho(\mathbf{x}) d(\mathbf{x}, \mathbf{x}_i) d\mathbf{x}$, where $\rho(\mathbf{x})$ is the density map that can be user designed. To minimize $E(X)$, we employ Lloyd's method [3] by iteratively updating the sites to the centers of their corresponding Voronoi cell. In each iteration the center of each cell is projected back on S. Fig. 3 shows two 2D examples under uniform and non-uniform distributions.



(a) (b)

Figure 3: *Illustration of 2D CVTs governed density map (deep color indicates high density). Two 2D examples are respectively under uniform (a) and non-uniform (b) distributions.*

B-Spline controlled hollowing. To generate the smooth pore in fine scale, we generate the restricted B-Spline curve in each Voronoi cell, similar to [4]. We first re-scale the cell ψ_i via width r, which implies the hollowing ratio. Then, we take the new cell vertices as the control points and use them to construct a closed B-Spline curve of degree three. We sample the closed curve and project the sampled points on S. We split and remesh the original triangular mesh in the intersection area between the projected curve and S, then we could get rid of the intersection to finally obtain a smooth, hollowed Voronoi cell. Once getting the final mesh for all cells, we extrude it along the surface normal direction to obtain a solid fabricable model. The hollowing process for each Voronoi cell is illustrated in Fig. 4.

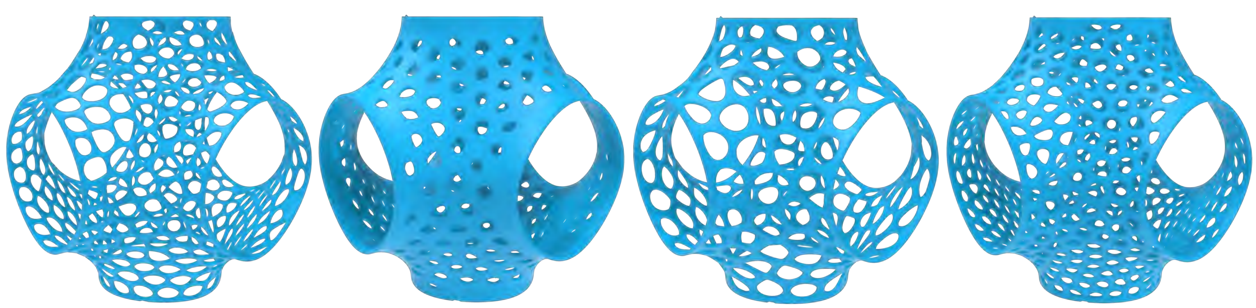


(a) (b) (c) (d)

Figure 4: *Hollowing process in a Voronoi cell. (a) Scale down the cell by r and get new vertices (white dots). (b) Generate a close B-Spline curve (blue line), and project it onto the original mesh (red line). (c) Remesh the mesh considering the B-Spline curve. (d) Hollow the cell.*

Results

We show results based on different types of minimal surfaces and the two parameters, width r and the number of sites n. Fig. 5 shows various scale and density of Voronoi cells with width r from 1mm to 4mm and n from 200 to 600 on a $100mm^3$ P-type TPMS unit cell. Fig. 6 shows results on regular TPMS like P, G and D types and other distorted TPMS. More results can be found in Fig. 7.

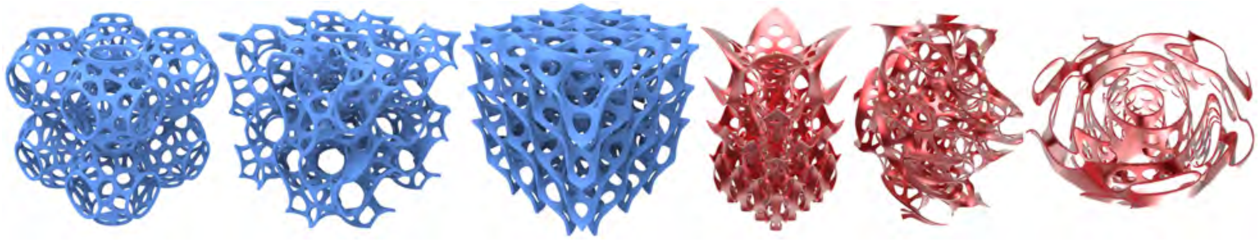


(a) (b) (c) (d)

Figure 5: *Various width $r(mm)$ and number n to control the scale and density of Voronoi cells on a P-type TPMS unit cell, (a) $r = 1, n = 400$, (b) $r = 4, n = 400$, (c) $r = 2.5, n = 200$, (d) $r = 2.5, n = 600$.*

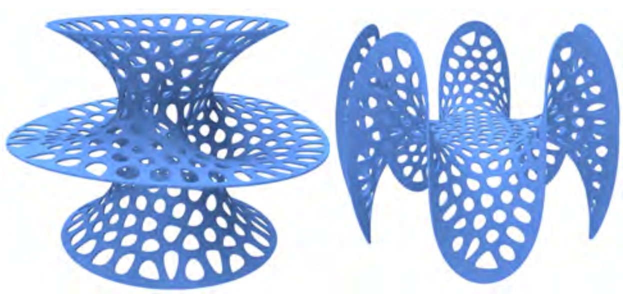
Conclusion

We have proposed a novel framework for modeling porous structure in a given boundary. Our bi-scale porous structures possess the properties like aesthetics, lightweight, boundary smoothness and ventilation. The large scale structure is created by triply periodic minimal surface bounded by the given shape. The fine scale pores

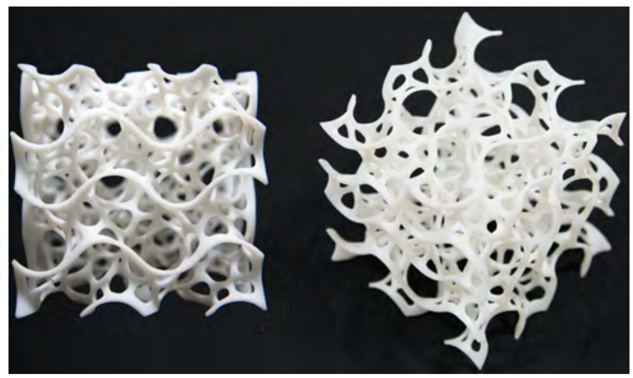


(a) (b) (c) (d) (e) (f)

Figure 6: *The bi-scale porous structures on regular TPMS: (a) P surface, (b) G surface, (c) D surface, and porous structures on distorted TPMS: (d-f).*



(a) *Bi-scale porous structures on other minimal surfaces. Costa surface (Left); Enneper surface (Right).*



(b) *The bi-scale porous structure fabricated by an SLA-3D printer, photographed from two different views.*

Figure 7: *More results.*

are distributed based on centroidal Voronoi tessellation and controlled by B-Spline curve in each cell. To our knowledge, this is the first attempt to combine these techniques for generating porous structures.

Acknowledgements

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